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Electroweak-boson hadroproduction at large transverse momentum: factorization, resummation, and NNLO corrections

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Abstract

We study the resummation of singular distributions for electroweak-boson production at large transverse momentum in hadronic collisions. We describe the factorization properties of the cross section near partonic threshold and construct the resummed cross section at next-to-leading logarithmic accuracy in moment space in terms of soft gluon anomalous dimensions. We present full analytical results for the expansion of the resummed cross section up to next-to-next-to-leading order. Our results can be applied to W , Z , and virtual γ production at hadron colliders.

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1 Introduction

The study of electroweak-boson production in hadron colliders provides useful tests of the standard model and estimates of backgrounds to new physics. For example, $Wb\bar{b}$ production is the principal background to the associated Higgs boson production, $p\bar{p} \rightarrow H(\rightarrow b\bar{b})W$, at the Tevatron [1]. The complete calculation of the next-to-leading-order (NLO) cross section for W , Z , and virtual γ production at large transverse momentum in hadron collisions has been presented in Refs. [2, 3], following earlier results for the non-singlet cross section in Ref. [4].

The calculation of hard scattering cross sections near the edges of phase space (partonic threshold), such as electroweak-boson production at high transverse momentum, involves corrections from the emission of soft gluons from the partons in the process. At each order in perturbation theory one encounters large logarithms that arise from incomplete cancellations near partonic threshold between graphs with real emission and virtual graphs. These threshold corrections exponentiate and have been resummed explicitly at next-to-leading logarithmic accuracy for a number of processes including heavy quark [5, 6, 7, 8, 9, 10], dijet [11, 9], and direct photon production [12, 13, 14], using general techniques developed originally for Drell-Yan production [15, 16]. For a review see Ref. [17].

In this paper we study electroweak-boson production at large transverse momentum in hadronic collisions where these considerations are of relevance. Following Ref. [12], we discuss in Section 2 the factorization properties of the single-particle inclusive cross section [18, 5, 11, 12] and identify its singular behavior near threshold. We then proceed to resum the leading (LL) and next-to-leading (NLL) logarithms explicitly to all orders in perturbation theory. In Section 3 we provide full analytical results for the expansion of the resummed cross section at NLO and at next-to-next-to-leading order (NNLO). Our NLO expansion agrees near partonic threshold with the exact NLO calculations in Refs. [2, 3] while our NNLO results provide new predictions.

2 Factorization and resummation

For the hadronic production of an electroweak boson V of mass m_V , where $V = W, Z, \gamma^*$,

$$h_A(P_A) + h_B(P_B) \longrightarrow V(Q) + X, \quad (2.1)$$

we write the factorized single-particle cross section as

$$\begin{aligned} E_Q \frac{d\sigma_{h_A h_B \rightarrow V(Q)+X}}{d^3Q} &= \sum_f \int dx_1 dx_2 \phi_{f_a/h_A}(x_1, \mu_F^2) \phi_{f_b/h_B}(x_2, \mu_F^2) \\ &\times E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow V(Q)+X}}{d^3Q}(s, t, u, Q, \mu_F, \alpha_s(\mu_R^2)) \end{aligned} \quad (2.2)$$

where $E_Q = Q^0$, $\phi_{f/h}$ is the parton distribution for parton f in hadron h , and $\hat{\sigma}$ is the perturbative cross section. The initial-state collinear singularities are factored into the ϕ 's at factorization scale μ_F , while μ_R is the renormalization scale.

At the parton level, the subprocesses for the production of an electroweak boson and a parton are

$$\begin{aligned} q(p_a) + g(p_b) &\longrightarrow q(p_c) + V(Q), \\ q(p_a) + \bar{q}(p_b) &\longrightarrow g(p_c) + V(Q). \end{aligned} \quad (2.3)$$

The hadronic and partonic kinematic invariants in the process are

$$\begin{aligned} S &= (P_A + P_B)^2, \quad T = (P_A - Q)^2, \quad U = (P_B - Q)^2, \quad S_2 = S + T + U - Q^2, \\ s &= (p_a + p_b)^2, \quad t = (p_a - Q)^2, \quad u = (p_b - Q)^2, \quad s_2 = s + t + u - Q^2, \end{aligned} \quad (2.4)$$

where S_2 and s_2 are the invariant masses of the system recoiling against the electroweak boson at the hadron and parton levels, respectively. $s_2 = (p_a + p_b - Q)^2$ parametrizes the inelasticity of the parton scattering: for one-parton production, $s_2 = 0$. Since x_i is the initial-parton momentum fraction, defined by $p_a = x_1 P_A$ and $p_b = x_2 P_B$, hadronic and partonic kinematic invariants are related by $s = x_1 x_2 S$, $t - Q^2 = x_1(T - Q^2)$, and $u - Q^2 = x_2(U - Q^2)$.

In general, $\hat{\sigma}$ includes distributions with respect to s_2 at n th order in α_s of the type

$$\left[\frac{\ln^m(s_2/Q^2)}{s_2} \right]_+, \quad m \leq 2n - 1, \quad (2.5)$$

defined by their integral with any smooth function f by

$$\int_0^{Q^2} ds_2 f(s_2) \left[\frac{\ln^m(s_2/Q^2)}{s_2} \right]_+ = \int_0^{Q^2} ds_2 \frac{\ln^m(s_2/Q^2)}{s_2} [f(s_2) - f(0)]. \quad (2.6)$$

These plus distributions are the remnants of cancellations between soft and virtual contributions to the cross section.

We now consider the partonic cross section, with the colliding hadrons in Eq. (2.2) replaced by partons. To organize the plus distributions in $\hat{\sigma}$, we introduce a refactorization [5, 11, 12] using new functions $\psi_{f/f}$ and J that describe the dynamics of partons moving collinearly to the incoming and outgoing partons respectively, and functions H and S which describe respectively the dynamics of hard partons and of soft partons which are not collinear to $\psi_{f/f}$ and J . This factorization is shown in Fig. 1 in cut diagram notation, showing contributions from the amplitude and its complex conjugate, with $H = hh^*$. At partonic threshold we find the relation

$$\begin{aligned} \frac{S_2}{S} &\simeq -(1-x_1)\frac{u}{s} - (1-x_2)\frac{t}{s} + \frac{s_2}{s} \\ &\equiv w_1 \left(\frac{-u}{s} \right) + w_2 \left(\frac{-t}{s} \right) + w_J + w_s, \end{aligned} \quad (2.7)$$

where in the second line of the equation the weights w identify the contributions of the functions in the refactorized cross section. At fixed S_2 , the partonic cross section factorizes into

$$\begin{aligned} E_Q \frac{d\sigma_{f_a f_b \rightarrow V}}{d^3 Q} &= H \int dw_1 dw_2 dw_J dw_s \psi_{f_a/f_a}(w_1) \psi_{f_b/f_b}(w_2) J(w_J) S(w_s Q/\mu_F) \\ &\times \delta \left(\frac{S_2}{S} - w_1 \left(\frac{-u}{s} \right) - w_2 \left(\frac{-t}{s} \right) - w_J - w_s \right). \end{aligned} \quad (2.8)$$

If we take moments of the above equation, with N the moment variable, we can write the partonic cross section as

$$\begin{aligned} \int \frac{dS_2}{S} e^{-NS_2/S} E_Q \frac{d\sigma_{f_a f_b \rightarrow V}}{d^3 Q} &= H \int dw_1 e^{-N_1 w_1} \psi_{f_a/f_a}(w_1) \\ &\times \int dw_2 e^{-N_2 w_2} \psi_{f_b/f_b}(w_2) \int dw_J e^{-N w_J} J(w_J) \int dw_s e^{-N w_s} S(w_s Q/\mu_F) \\ &\equiv \tilde{\psi}_{f_a/f_a}(N_1) \tilde{\psi}_{f_b/f_b}(N_2) \tilde{J}(N) H \tilde{S}(Q/(N\mu_F)), \end{aligned} \quad (2.9)$$

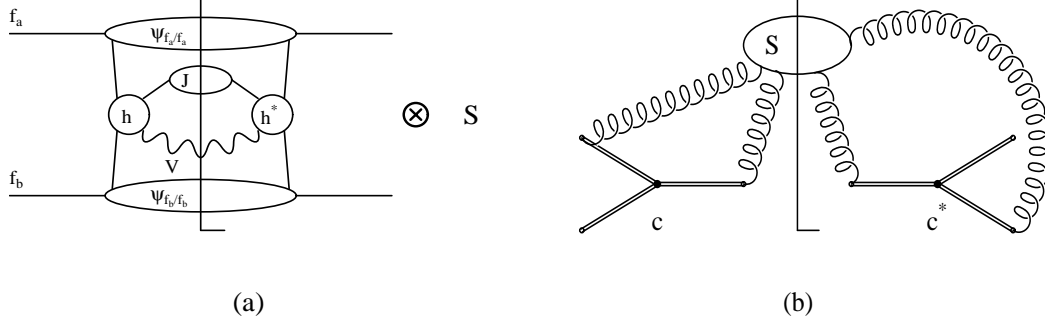


Fig. 1. (a) Factorization for $V + \text{jet}$ production near partonic threshold. (b) The soft-gluon function S , in which the vertices c link eikonal lines representing the partons in the process.

with $N_1 = N(-u/s)$ and $N_2 = N(-t/s)$ for either partonic subprocess.

Taking moments of Eq. (2.2) with the incoming hadrons replaced by partons, and using the first line of Eq. (2.7), we also have the relation

$$\begin{aligned} \int \frac{dS_2}{S} e^{-NS_2/S} E_Q \frac{d\sigma_{f_a f_b \rightarrow V}}{d^3 Q} &= \int dx_1 e^{-N_1(1-x_1)} \phi_{f_a/f_a}(x_1, \mu_F^2) \\ &\times \int dx_2 e^{-N_2(1-x_2)} \phi_{f_b/f_b}(x_2, \mu_F^2) \int \frac{ds_2}{s} e^{-Ns_2/s} E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow V}(s_2)}{d^3 Q} \\ &\equiv \tilde{\phi}_{f_a/f_a}(N_1) \tilde{\phi}_{f_b/f_b}(N_2) E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow V}(N)}{d^3 Q}. \end{aligned} \quad (2.10)$$

Note that $s_2/s = s_2/S$ up to quadratic terms in $(1-x_1)$ and/or $(1-x_2)$ and/or s_2 . Comparing Eqs. (2.10) and (2.9) we then may solve for the moments of the perturbative cross section $\hat{\sigma}$:

$$E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow V}(N)}{d^3 Q} = \frac{\tilde{\psi}_{f_a/f_a}(N_1) \tilde{\psi}_{f_b/f_b}(N_2)}{\tilde{\phi}_{f_a/f_a}(N_1) \tilde{\phi}_{f_b/f_b}(N_2)} \tilde{J}(N) H \tilde{S}(Q/(N\mu_F)). \quad (2.11)$$

The plus distributions in $\hat{\sigma}$ produce, under moments, powers of $\ln N$ as high as $\ln^{2n} N$. The LL corrections are included in ψ/ϕ and J , while the NLL corrections are included in ψ/ϕ , J , and S .

The resummation of the N -dependence of the jet and soft functions in Eq. (2.11) depends on their renormalization properties [5, 11]. The factor $\tilde{\psi}/\tilde{\phi}$ is universal between electroweak and QCD-induced hard processes and its resummation was first done in the context of the Drell-Yan process [15, 16].

It contributes an enhancement to the cross section, while the final-state jet \tilde{J} gives a negative contribution [11, 12]. The UV divergences induced by factorization in the hard and soft functions cancel against each other since there are no additional UV divergences aside from those already removed through the usual renormalization procedure. The renormalization of the hard and soft functions can be written as

$$H^{(0)} = \prod_{i=a,b} Z_i^{-1} Z_S^{-1} H(Z_S^*)^{-1}, \quad S^{(0)} = Z_S^* S Z_S, \quad (2.12)$$

where Z_S is the renormalization constant of the soft function, and Z_i is the renormalization constant of the i th incoming partonic field. Then S satisfies the renormalization group equation

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) S = -2(\text{Re } \Gamma_S) S, \quad \Gamma_S(g) = -\frac{g}{2} \frac{\partial}{\partial g} \text{Res}_{\epsilon \rightarrow 0} Z_S(g, \epsilon), \quad (2.13)$$

where Γ_S is the soft anomalous dimension. We may then evolve S from the scale Q/N to μ_F as

$$\tilde{S} \left(\frac{Q}{\mu_F N}, \alpha_s(\mu_F^2) \right) = \tilde{S} \left(1, \alpha_s(Q^2/N^2) \right) \exp \left[\int_{\mu_F}^{Q/N} \frac{d\mu'}{\mu'} 2\text{Re} \Gamma_S \left(\alpha_s(\mu'^2) \right) \right]. \quad (2.14)$$

The resummation of the N -dependence of each of the functions in the refactored cross section, Eq. (2.11), in the $\overline{\text{MS}}$ factorization scheme, then gives [5, 11, 12]

$$\begin{aligned} E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow V}(N)}{d^3 Q} &= \exp \left\{ \sum_{i=a,b} \left[E_{(f_i)}(N_i) \right. \right. \\ &\quad \left. \left. - 2 \int_{\mu_F}^{2p_i \cdot \zeta} \frac{d\mu'}{\mu'} \left[\gamma_{f_i}(\alpha_s(\mu'^2)) - \gamma_{f_i f_i}(N_i, \alpha_s(\mu'^2)) \right] \right] \right\} \\ &\times \exp \left\{ E'_{(J)}(N) \right\} H \left(\alpha_s(\mu_F^2) \right) \\ &\times \tilde{S} \left(1, \alpha_s(Q^2/N^2) \right) \exp \left[\int_{\mu_F}^{Q/N} \frac{d\mu'}{\mu'} 2\text{Re} \Gamma_S \left(\alpha_s(\mu'^2) \right) \right], \quad (2.15) \end{aligned}$$

with $\zeta^\mu = p_c^\mu/Q$. The incoming parton-jet anomalous dimensions [11, 10], defined through

$$\mu \frac{d\tilde{\psi}_{f/f}(N, Q/\mu, \epsilon)}{d\mu} = 2\gamma_f(\alpha_s(\mu^2)) \tilde{\psi}_{f/f}(N, Q/\mu, \epsilon) \quad (2.16)$$

and

$$\mu \frac{d\tilde{\phi}_{f/f}(N, \mu^2, \epsilon)}{d\mu} = 2\gamma_{ff}(N, \alpha_s(\mu^2)) \tilde{\phi}_{f/f}(N, \mu^2, \epsilon), \quad (2.17)$$

are given by

$$\begin{aligned} \gamma_q &= \frac{3}{4} C_F \frac{\alpha_s}{\pi}; & \gamma_{qq} &= -\left(\ln N - \frac{3}{4}\right) C_F \frac{\alpha_s}{\pi}, \\ \gamma_g &= \frac{\beta_0}{4} \frac{\alpha_s}{\pi}; & \gamma_{gg} &= -\left(C_A \ln N - \frac{\beta_0}{4}\right) \frac{\alpha_s}{\pi}, \end{aligned} \quad (2.18)$$

for quark and gluon jets, respectively. $\beta_0 = (11C_A - 2n_f)/3$ is the one-loop coefficient of the β function, with n_f the number of quark flavors. The exponents for the incoming jets are [5, 11, 12]

$$\begin{aligned} E_{(fi)}(N_i) &= -\int_0^1 dz \frac{z^{N_i-1} - 1}{1-z} \left\{ \int_{(1-z)^2}^1 \frac{d\lambda}{\lambda} A_{(fi)} \left[\alpha_s \left((2p_i \cdot \zeta)^2 \lambda \right) \right] \right. \\ &\quad \left. + \frac{1}{2} \nu^{(fi)} \left[\alpha_s \left((2p_i \cdot \zeta)^2 (1-z)^2 \right) \right] \right\}. \end{aligned} \quad (2.19)$$

At next-to-leading order accuracy in $\ln(N)$, we need $\nu^{(f)} = 2C_f \alpha_s/\pi$ and $A_{(f)}(\alpha_s) = C_f \left(\alpha_s/\pi + (K/2) (\alpha_s/\pi)^2 \right)$, with $C_f = C_F$ (C_A) for an incoming quark (gluon), and $K = C_A(67/18 - \pi^2/6) - 5n_f/9$.

The exponent for the final-state jet is [11, 12]

$$\begin{aligned} E'_{(J)}(N) &= \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left\{ \int_{(1-z)^2}^{(1-z)} \frac{d\lambda}{\lambda} A_{(J)} \left[\alpha_s(\lambda Q^2) \right] \right. \\ &\quad \left. + B'_{(J)} \left[\alpha_s \left((1-z) Q^2 \right) \right] + B''_{(J)} \left[\alpha_s \left((1-z)^2 Q^2 \right) \right] \right\} \end{aligned} \quad (2.20)$$

where²

$$B'_{(q)} = \frac{\alpha_s}{\pi} \left(-\frac{3}{4} \right) C_F, \quad B''_{(q)} = \frac{\alpha_s}{\pi} C_F [\ln(2\nu_q) - 1],$$

²Note that Eq. (2.20) corrects the arguments of α_s in B' in Eqs. (11) and (12) of Ref. [12].

$$B'_{(g)} = \frac{\alpha_s}{\pi} \left(-\frac{\beta_0}{4} \right), \quad B''_{(g)} = \frac{\alpha_s}{\pi} C_A [\ln(2\nu_g) - 1]. \quad (2.21)$$

The ν_i terms are gauge dependent. They are defined by $\nu_i \equiv (\beta_i \cdot n)^2/|n|^2$, where $\beta_i = p_i \sqrt{2/s}$ are the particle velocities and n is the gauge vector, chosen so that $p_i \cdot \zeta = p_i \cdot n$ for $i = a, b$ [12]. In our calculations we use Feynman rules for eikonal diagrams in axial gauge (resummation calculations have also been done in covariant gauge [19]).

The soft anomalous dimensions are calculated explicitly by evaluating one-loop vertex corrections [5, 17]. For the $qg \rightarrow qV$ channel in the kinematics (2.4) we find [12, 17]

$$\begin{aligned} \Gamma_S^{qg \rightarrow qV} = & \frac{\alpha_s}{2\pi} \left\{ C_F \left[2 \ln \left(\frac{-u}{s} \right) - \ln(4\nu_{q_a} \nu_{q_c}) + 2 \right] \right. \\ & \left. + C_A \left[\ln \left(\frac{t}{u} \right) - \ln(2\nu_g) + 1 - \pi i \right] \right\}. \end{aligned} \quad (2.22)$$

The soft anomalous dimension for $q\bar{q} \rightarrow gV$ is [12, 17]

$$\Gamma_S^{q\bar{q} \rightarrow gV} = \frac{\alpha_s}{2\pi} \left\{ C_F [-\ln(4\nu_q \nu_{\bar{q}}) + 2 - 2\pi i] + C_A \left[\ln \left(\frac{tu}{s^2} \right) - \ln(2\nu_g) + 1 + \pi i \right] \right\}. \quad (2.23)$$

Eqs. (2.22) and (2.23) coincide with the corresponding soft anomalous dimensions for direct photon production [12, 17]. Substituting Eqs. (2.19) through (2.23) in the resummed cross section, Eq. (2.15), we see that at NLL accuracy all the gauge-dependent terms cancel out in the exponent, for both the $qg \rightarrow qV$ channel and the $q\bar{q} \rightarrow gV$ channel.

We can rewrite the resummed cross section in a form which is more convenient for the calculation of the fixed-order expansions. Using the renormalization group behavior of the product HS from Eq. (2.12),

$$\mu \frac{d}{d\mu} \ln [H(\mu)S(Q/(N\mu))] = -2 [\gamma_a(\alpha_s(\mu^2)) + \gamma_b(\alpha_s(\mu^2))] , \quad (2.24)$$

and the relation

$$H(\alpha_s(Q^2)) = H(\alpha_s(\mu_R^2)) \exp \left[\int_{\mu_R}^Q 2 \frac{d\mu'}{\mu'} \beta(\alpha_s(\mu^2)) \right], \quad (2.25)$$

with $\beta(\alpha_s) \equiv \mu d \ln g / d\mu = -\beta_0 \alpha_s / 4\pi + \dots$, we can write Eq. (2.15) as

$$\begin{aligned}
E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow V}(N)}{d^3 Q} &= H(\alpha_s(\mu_R^2)) \exp \left[2 \int_{\mu_R}^Q \frac{d\mu'}{\mu'} \beta(\alpha_s(\mu'^2)) \right] \\
&\times \exp \left\{ \sum_{i=a,b} \left[E_{(f_i)}(N_i) - 2 \int_{\mu_F}^{2p_i \cdot \zeta} \frac{d\mu'}{\mu'} \left[\gamma_{f_i}(\alpha_s(\mu'^2)) - \gamma_{f_i f_i}(N_i, \alpha_s(\mu'^2)) \right] \right] \right\} \\
&\times \exp \{ E'_{(J)}(N) \} \exp \left[2 \int_{\mu_F}^Q \frac{d\mu'}{\mu'} \left(\gamma_a(\alpha_s(\mu'^2)) + \gamma_b(\alpha_s(\mu'^2)) \right) \right] \\
&\times \tilde{S}(1, \alpha_s(Q^2/N^2)) \exp \left[\int_Q^{Q/N} \frac{d\mu'}{\mu'} 2\text{Re}\Gamma_S(\alpha_s(\mu'^2)) \right]. \quad (2.26)
\end{aligned}$$

3 Next-to-next-to-leading order expansion of the resummed cross section

In this section, we expand the NLL resummed cross section up to NNLO, invert back from moment space to momentum space, and perform a comparison with the NLO results of Ref. [2, 3]. At the parton level, the subprocesses for the production of an electroweak boson and a parton are given in Eq. (2.3).

We can write the NLO corrections from Eq. (2.26) for $qg \rightarrow qV$ in single-particle inclusive kinematics at NLL accuracy as

$$\begin{aligned}
E_Q \frac{d\hat{\sigma}_{qg \rightarrow qV}^{\text{NLO}}}{d^3 Q} &= \sigma_{qg \rightarrow qV}^B \frac{\alpha_s(\mu_R^2)}{\pi} \left\{ (C_F + 2C_A) \left[\frac{\ln(s_2/Q^2)}{s_2} \right]_+ \right. \\
&- \left[(C_F + C_A) \ln \left(\frac{\mu_F^2}{Q^2} \right) + \frac{3}{4} C_F + C_A \ln \left(\frac{tu}{sQ^2} \right) \right] \left[\frac{1}{s_2} \right]_+ \\
&+ \delta(s_2) \left[\ln \left(\frac{\mu_F^2}{Q^2} \right) \left(-\frac{\beta_0}{4} + C_F \left(\ln \left(\frac{-u}{Q^2} \right) - \frac{3}{4} \right) + C_A \ln \left(\frac{-t}{Q^2} \right) \right) \right. \\
&\left. \left. + \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{Q^2} \right) \right] \right\}. \quad (3.1)
\end{aligned}$$

Here the Born term is

$$\sigma_{qg \rightarrow qV}^B = \frac{\alpha \alpha_s(\mu_R^2) C_F}{s(N_c^2 - 1)} A^{qg} \sum_f (|L_{ff_a}|^2 + |R_{ff_a}|^2), \quad (3.2)$$

$$A^{qg} = - \left(\frac{s}{t} + \frac{t}{s} + \frac{2uQ^2}{st} \right),$$

with L and R the left- and right-handed couplings of the electroweak boson to the quark line, f the quark flavor and \sum_f the sum over the flavors allowed by the CKM mixing and by the energy threshold. We choose for the L, R couplings the conventions of Ref. [3]. The Born differential cross section is $E_Q d\sigma_{qg \rightarrow qV}^B / d^3Q = \sigma_{qg \rightarrow qV}^B \delta(s_2)$.

We have kept the factorization and renormalization scales separate. The expansion gives all $[\ln(s_2/Q^2)/s_2]_+$ and $[1/s_2]_+$ terms but only scale-dependent $\delta(s_2)$ terms.

Our one-loop expansion can be compared with the exact NLO cross section [3] in the proximity of the partonic threshold, $s_2 \rightarrow 0$, i.e. with the singular terms of the type (2.5) in the contribution of the subprocess $qg \rightarrow qVg$ to the cross section for $qg \rightarrow qV$. We find full agreement³ with the results of Ref. [3].

Then we expand the resummed cross section, Eq. (2.26), for $qg \rightarrow qV$ to two-loop order and compute the NNLO corrections in single-particle inclusive kinematics at NLL accuracy, including all the factorization and renormalization scale dependent terms. We obtain

$$\begin{aligned} E_Q \frac{d\hat{\sigma}_{qg \rightarrow qV}^{\text{NNLO}}}{d^3Q} &= \sigma_{qg \rightarrow qV}^B \left(\frac{\alpha_s(\mu_R^2)}{\pi} \right)^2 \left\{ \frac{1}{2} (C_F + 2C_A)^2 \left[\frac{\ln^3(s_2/Q^2)}{s_2} \right]_+ \right. \\ &+ \left[-\frac{3}{2} (C_F + 2C_A) \left((C_F + C_A) \ln \left(\frac{\mu_F^2}{Q^2} \right) + \frac{3}{4} C_F + C_A \ln \left(\frac{tu}{sQ^2} \right) \right) \right. \\ &\quad \left. \left. - \frac{\beta_0}{2} \left(\frac{C_F}{4} + C_A \right) \right] \left[\frac{\ln^2(s_2/Q^2)}{s_2} \right]_+ \right. \\ &+ \ln \left(\frac{\mu_F^2}{Q^2} \right) \left[(C_F + C_A)^2 \ln \left(\frac{\mu_F^2}{Q^2} \right) + \frac{3}{4} C_F^2 + 2C_A(C_F + C_A) \ln \left(\frac{tu}{sQ^2} \right) \right. \\ &\quad \left. + (C_F + 2C_A) \left(C_F \ln \left(\frac{-u}{Q^2} \right) + C_A \ln \left(\frac{-t}{Q^2} \right) - \frac{\beta_0}{4} \right) \right] \left[\frac{\ln(s_2/Q^2)}{s_2} \right]_+ \\ &\left. + \ln \left(\frac{\mu_R^2}{Q^2} \right) \frac{\beta_0}{2} (C_F + 2C_A) \left[\frac{\ln(s_2/Q^2)}{s_2} \right]_+ \right\} \end{aligned}$$

³Note that Gonsalves et al. [3] use “A+” distributions, which are related to our “+” distributions by $[\ln^n(s_2/Q^2)/s_2]_+ = [\ln^n(s_2/Q^2)/s_2]_{A+} + (1/(n+1)) \ln^{n+1}(A/Q^2) \delta(s_2)$.

$$\begin{aligned}
& + \ln^2 \left(\frac{\mu_F^2}{Q^2} \right) (C_F + C_A) \left[C_F \left(-\ln \left(\frac{-u}{Q^2} \right) + \frac{3}{4} \right) - C_A \ln \left(\frac{-t}{Q^2} \right) + \frac{3\beta_0}{8} \right] \left[\frac{1}{s_2} \right]_+ \\
& - \frac{\beta_0}{2} (C_F + C_A) \ln \left(\frac{\mu_R^2}{Q^2} \right) \ln \left(\frac{\mu_F^2}{Q^2} \right) \left[\frac{1}{s_2} \right]_+ \Big\} . \tag{3.3}
\end{aligned}$$

Our expansion gives all $[\ln^3(s_2/Q^2)/s_2]_+$ and $[\ln^2(s_2/Q^2)/s_2]_+$ terms but only scale-dependent terms for $[\ln(s_2/Q^2)/s_2]_+$ and $[1/s_2]_+$. However we can derive all the $[\ln(s_2/Q^2)/s_2]_+$ terms by matching with the exact NLO cross section⁴. Including the $\mu_{R,F}$ -dependent terms given above, the full $[\ln(s_2/Q^2)/s_2]_+$ terms are

$$\begin{aligned}
& \sigma_{qg \rightarrow qV}^B \left(\frac{\alpha_s(\mu_R^2)}{\pi} \right)^2 \left\{ \frac{1}{2A^{qg}} (C_F + 2C_A) [B_1^{qg} + B_2^{qg} n_f + C_1^{qg} + C_2^{qg} n_f \right. \\
& \quad \left. + B_3^{qg} \frac{(L_{ffa} - R_{ffa}) \sum_f (L_{ff} - R_{ff})}{\sum_f (|L_{ffa}|^2 + |R_{ffa}|^2)} \right] \\
& \quad + \left[(C_F + C_A) \ln \left(\frac{\mu_F^2}{Q^2} \right) + \frac{3}{4} C_F + C_A \ln \left(\frac{tu}{sQ^2} \right) \right]^2 \\
& \quad + (C_F + 2C_A) \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{Q^2} \right) + \frac{1}{2} K(C_F + 2C_A) - \zeta_2 (C_F + 2C_A)^2 \\
& \quad \left. + \beta_0 \frac{C_F}{4} \left(\frac{3}{4} - \ln \left(\frac{s}{Q^2} \right) \right) + \beta_0 \frac{C_A}{2} \ln \left(\frac{tu}{sQ^2} \right) \right\} \left[\frac{\ln(s_2/Q^2)}{s_2} \right]_+ , \tag{3.4}
\end{aligned}$$

with $\zeta_2 = \pi^2/6$, and with B_1^{qg} , B_2^{qg} , B_3^{qg} , C_1^{qg} , and C_2^{qg} as given in the Appendix of Ref. [3] but without the renormalization counterterms and using $f_A \equiv \ln(A/Q^2) = 0$.

Next, we consider the $q\bar{q} \rightarrow gV$ partonic subprocess, and compute the NLO corrections from Eq. (2.26) in single-particle inclusive kinematics at NLL accuracy,

$$\begin{aligned}
E_Q \frac{d\hat{\sigma}_{q\bar{q} \rightarrow gV}^{\text{NLO}}}{d^3Q} &= \sigma_{q\bar{q} \rightarrow gV}^B \frac{\alpha_s(\mu_R^2)}{\pi} \left\{ (4C_F - C_A) \left[\frac{\ln(s_2/Q^2)}{s_2} \right]_+ \right. \\
&\quad \left. - \left[2C_F \ln \left(\frac{\mu_F^2}{Q^2} \right) + (2C_F - C_A) \ln \left(\frac{tu}{sQ^2} \right) + \frac{\beta_0}{4} \right] \left[\frac{1}{s_2} \right]_+ \right\}
\end{aligned}$$

⁴In Eq. (2.12) of Ref. [3] there is a typo: the term C_2^{qg} should be replaced by $\delta(s_2)C_2^{qg}n_f + C_3^{qg}$.

$$+ \delta(s_2) \left[\ln \left(\frac{\mu_F^2}{Q^2} \right) C_F \left(\ln \left(\frac{tu}{Q^4} \right) - \frac{3}{2} \right) + \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{Q^2} \right) \right] \}. \quad (3.5)$$

Here the Born term is

$$\begin{aligned} \sigma_{q\bar{q} \rightarrow gV}^B &= \frac{\alpha \alpha_s (\mu_R^2) C_F}{s N_c} A^{q\bar{q}} (|L_{f_b f_a}|^2 + |R_{f_b f_a}|^2), \\ A^{q\bar{q}} &= \frac{u}{t} + \frac{t}{u} + \frac{2Q^2 s}{tu}, \end{aligned} \quad (3.6)$$

and the Born differential cross section is $E_Q d\sigma_{q\bar{q} \rightarrow gV}^B / d^3Q = \sigma_{q\bar{q} \rightarrow gV}^B \delta(s_2)$.

The one-loop term can be compared with the singular terms of the type (2.5) in the contribution of the subprocesses $q\bar{q} \rightarrow gVg$ and $q\bar{q} \rightarrow Vq\bar{q}$ to the cross section for $q\bar{q} \rightarrow gV$. We are in agreement with Ref. [3].

The NNLO corrections from Eq. (2.26) for $q\bar{q} \rightarrow gV$ in single-particle inclusive kinematics at NLL accuracy, including all the factorization and renormalization scale dependent terms, are

$$\begin{aligned} E_Q \frac{d\hat{\sigma}_{q\bar{q} \rightarrow gV}^{\text{NNLO}}}{d^3Q} &= \sigma_{q\bar{q} \rightarrow gV}^B \left(\frac{\alpha_s(\mu_R^2)}{\pi} \right)^2 \left\{ \frac{1}{2} (4C_F - C_A)^2 \left[\frac{\ln^3(s_2/Q^2)}{s_2} \right]_+ \right. \\ &+ \left[-\frac{3}{2} (4C_F - C_A) \left(2C_F \ln \left(\frac{\mu_F^2}{Q^2} \right) + (2C_F - C_A) \ln \left(\frac{tu}{sQ^2} \right) \right) \right. \\ &\quad \left. \left. - \frac{\beta_0}{2} \left(5C_F - \frac{3}{2} C_A \right) \right] \left[\frac{\ln^2(s_2/Q^2)}{s_2} \right]_+ \right. \\ &+ \ln \left(\frac{\mu_F^2}{Q^2} \right) \left[C_F \left(4C_F \ln \left(\frac{\mu_F^2}{Q^2} \right) + (4C_F - C_A) \left(\ln \left(\frac{tu}{Q^4} \right) - \frac{3}{2} \right) \right. \right. \\ &\quad \left. \left. + 4(2C_F - C_A) \ln \left(\frac{tu}{sQ^2} \right) \right) + C_F \beta_0 \right] \left[\frac{\ln(s_2/Q^2)}{s_2} \right]_+ \\ &+ \ln \left(\frac{\mu_R^2}{Q^2} \right) \frac{\beta_0}{2} (4C_F - C_A) \left[\frac{\ln(s_2/Q^2)}{s_2} \right]_+ \\ &+ \ln^2 \left(\frac{\mu_F^2}{Q^2} \right) C_F \left[C_F \left(-2 \ln \left(\frac{tu}{Q^4} \right) + 3 \right) + \frac{\beta_0}{4} \right] \left[\frac{1}{s_2} \right]_+ \\ &\left. - \beta_0 C_F \ln \left(\frac{\mu_R^2}{Q^2} \right) \ln \left(\frac{\mu_F^2}{Q^2} \right) \left[\frac{1}{s_2} \right]_+ \right\}. \quad (3.7) \end{aligned}$$

Again, only scale-dependent terms for $[\ln(s_2/Q^2)/s_2]_+$ and $[1/s_2]_+$ are included in Eq. (3.7). The full $[\ln(s_2/Q^2)/s_2]_+$ terms are found by matching

with the exact NLO cross section,

$$\begin{aligned}
\sigma_{q\bar{q} \rightarrow gV}^B & \left(\frac{\alpha_s(\mu_R^2)}{\pi} \right)^2 \left\{ \frac{1}{2A^{q\bar{q}}} (4C_F - C_A) \left[B_1^{q\bar{q}} + C_1^{q\bar{q}} + (B_2^{q\bar{q}} + D_{aa}^{(0)}) n_f \right. \right. \\
& \quad \left. \left. + B_3^{q\bar{q}} \delta_{f_a f_b} \frac{(L_{f_a f_a} - R_{f_a f_a}) \sum_f (L_{ff} - R_{ff})}{(|L_{f_b f_a}|^2 + |R_{f_b f_a}|^2)} \right] \right. \\
& \quad + \left[2C_F \ln \left(\frac{\mu_F^2}{Q^2} \right) + (2C_F - C_A) \ln \left(\frac{tu}{sQ^2} \right) + \frac{\beta_0}{4} \right]^2 \\
& \quad + (4C_F - C_A) \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{Q^2} \right) + (4C_F - C_A) \frac{K}{2} + \frac{\beta_0^2}{16} - \zeta_2 (4C_F - C_A)^2 \\
& \quad \left. + \beta_0 \left(C_F - \frac{C_A}{2} \right) \ln \left(\frac{tu}{sQ^2} \right) - C_A \frac{\beta_0}{4} \ln \left(\frac{s}{Q^2} \right) \right\} \left[\frac{\ln(s_2/Q^2)}{s_2} \right]_+, \quad (3.8)
\end{aligned}$$

with $B_1^{q\bar{q}}$, $B_2^{q\bar{q}}$, $B_3^{q\bar{q}}$, $C_1^{q\bar{q}}$, and $D_{aa}^{(0)}$ as given in the Appendix of Ref. [3] but without the renormalization counterterms and using $f_A = 0$.

4 Conclusion

We have presented the resummed cross section at NLL accuracy for electroweak-boson production at large transverse momentum near partonic threshold. The expansion of the resummed cross section at NLO agrees with previous exact NLO results while the NNLO expansion provides new predictions at higher order. In future work we plan to study the numerical significance of resummation for W +jet production. Related studies for other processes have shown reduced factorization scale-dependence, expected on theoretical grounds [20], and increases over the NLO cross section.

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